**Investigating the Effects of Some Input Variables on the Operation of an Evaporator through Dynamics**

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**Abstract**

The mathematical model of an evaporator, showing the dynamic behavior of liquid height and system temperature, has been developed. The model was used to investigate the dynamics of the system by simulating the obtained nonlinear differential equations using the *ode45* command of MATLAB. Furthermore, using the realistic steady-state values obtained by solving the developed model with the aid of *fsolve* command of the same MATLAB, the system of transfer function model of the system was obtained via its state-space form. Moreover, the Simulink model of the system was developed using the obtained transfer functions of the model. Thereafter, the dynamics of the system was investigated by simulating the Simulink model through the application of input steps to the liquid feed flow rate and the steam temperature of the system. The results obtained from the development of the transfer function models and the simulations carried out when steps were applied to the input variables of the system revealed that the steam temperature had not any effect on the liquid level of the system as their transfer function was found to be zero, and the application of a step to steam temperature did not result into any change in the steady-state value of the liquid level. It was also discovered that a step change in liquid feed flow rate affected both the liquid level and the system temperature. Despite the fact that a step change in steam temperature could not affect the liquid height (level) of the evaporator, the system has been found to be a Multi-Input Multi-Output (MIMO) type because, at least, the liquid feed flow rate (one of the input variables) could result into changes in the two output variables (liquid height and system temperature) of the system in addition to the effect the steam temperature (the other input variable) had on the temperature of the evaporator.

**INTRODUCTION**

Evaporation is an engineering operation that is usually employed to remove a liquid from a solution, suspension or liquor by boiling off some of the liquid. It is usually treated as the separation of a liquid mixture into concentrate and vapor. This operation is usually carried out in a heating device called *evaporator*. Evaporator is made of a heat exchanger for heating up a solution and a means to separate vapor from the boiling solution.

Evaporators are often used in sugar making industry, food processing industries, and pulp and paper mill (Kroskhwitz and Howe-Grant, 1994). In order to study and understand the operation of an evaporator, one of the methods used is to develop the model of that particular evaporator. In order words, the operation of an evaporator can be studied via Mathematical Modeling.
Mathematical Modeling of an evaporator is usually developed from the principles of conservation of mass and energy (Yusuff et al., 2013). In an evaporator, heat is transferred from a heating medium to a solution. As the solution boils, material and heat are simultaneously transferred into a vapor phase. Mathematical representation of an evaporation process consists of an overall material balance, component material balance, energy balances for both liquid and heating medium and heat transfer equations.

Also, the design of effective control system is the main objective of an evaporator manufacturer. Process system control strategies have been predominantly employed in chemical, petrochemical, agrochemical and some other important process industries where safety, high production rate and product quality is of great concern (Benson, 1997; Culter and Ramaker, 1979; Austin and Bozin, 1996; Willis et al., 1992). In order to develop a good and robust model for an evaporator, it is necessary that it should, first, be well studied and understood. This is one of the reasons why there are a researches going on in this area these days. Besides, it has been deemed necessary to obtain a reliable model that can represent the operation of an evaporator that can be used conveniently for controlled purposes.

A lot of research works have been carried out on modeling and simulation of different kinds of evaporators. For instance, Miranda and Simpson (2005) investigated a phenomenological, stationary and dynamic model for a multiple effect evaporator for simulation and control purposes. The model took into account the variation of temperature and concentration. Furthermore, their studies suggested that the global heat transfer coefficient and the latent heat of vaporization were the most important parameters of the process. Campos and Lage (2001) studied the dynamic model of a direct contact evaporator. In their work, the developed model, which took into account heat and mass transfer during the bubble formation and ascension stages, was solved and the results of the coupled model were compared with experimental data. It was observed that only liquid height was simulated at transient condition. Lage and Hackenberg (1990) also investigated a dynamic model for the liquid phase which was coupled to the superheated bubble model developed by Queiroz (1990). The model showed good agreement with the ones reported in the literature (Queiroz, 1990; Queiroz and Hackenberg, 1997) when experimental gas holdups were used as input data. Based on the literature review carried out, it was discovered that very few researches have been carried out concerning using transfer function to simulate an evaporator. It was thus felt that this gap has to be closed.

Therefore, in this work, the transfer function models of an evaporator have been developed via its state-space model using its theoretical dynamic model. The developed transfer model was simulated by applying steps to its input variables (liquid feed flow rate and steam temperature) to obtain the responses of its output variables (liquid height and temperature of the system).

Methodology

The method adopted in this work was the modelling and simulation of the evaporator being considered. First and foremost, the model of the system was developed from first principle. Later on, its state space model, and, thereafter, its transfer function models were developed. The developed system of transfer function models was then simulated through the application of steps to the input variables of the evaporator.

System Description

The evaporator considered, shown in Figure 1, was fed by a stream $F_{in}$ at a temperature $T_{in}$. It was heated by steam-fed heating coils, with the steam pressure represented by $P_{steam}$. The pressure of the vapour in the evaporator was denoted as $P$. The evaporator product stream (a vapour) leaves the evaporator with a flow, $F_{out}$, and temperature, $T$. Equilibrium was assumed to be instantaneous, with the liquid temperature being equal to the vapour temperature $T$. The product stream passed through a valve, with the downstream pressure equal to $P_{out}$. The level of the liquid inside the evaporator was denoted by $h$. 
Figure 1. A schematic diagram of an evaporator

Model Development and Simulation

To develop the mathematical model that will be able to represent the evaporation operation occurring in the system described above, the following assumptions were made:

- The liquid in the tank is ideally mixed.
- Vapor-liquid equilibrium is instantaneous.
- The vapor does not exchange heat with the coil.
- $F_{\text{out}}$ depends on the square root of the pressure drop over the valve.
- The same temperature exists everywhere in the coil.
- The mass of vapor is ignorable compared to the mass of the liquid.
- All physical properties are constant in the operating range.
- The cross-sectional area of the vessel is constant.

The principles of conservation of mass (without any chemical reaction) can be written, mathematically, as,

\[
\left\{ \text{Rate of accumulation of material within a system} \right\} = \left\{ \text{Rate of material into the system} \right\} - \left\{ \text{Rate of material out of the system} \right\}
\]  

(1)

For the evaporator being considered, subject to the aforementioned assumptions and the principles stated in Equation (1) above,

\[
\rho A_e \frac{dh}{dt} = F_{\text{in}} - F_{\text{out}}
\]  

(2)

Since,

\[
F_{\text{out}} = c_v \sqrt{P - P_{\text{out}}}
\]  

(3)

Equation (2) becomes,
\[ \rho A \frac{dh}{dt} = F_{in} - c_v \sqrt{P - P_{out}} \]  

(4)

Also, the pressure difference of the system, \( P - P_{out} \), can be expressed in terms of liquid properties as:

\[ P - P_{out} = (-\Delta P) = \rho gh \]  

(5)

As such, Equation (3) becomes,

\[ \rho A \frac{dh}{dt} = F_{in} - c_v \sqrt{\rho gh} \]  

(6)

Then,

\[ \frac{dh}{dt} = \frac{F_{in} - c_v \sqrt{\rho gh}}{\rho A_v} \]  

(7)

Also, for the energy (enthalpy) balance,

\[
\begin{align*}
\text{Rate of accumulation of energy within a system} &= \text{Rate of energy into the system} - \text{Rate of energy out of the system} \\
\end{align*}
\]

(8)

So,

\[ \rho c_p A_v \frac{d(h[T - T_{ref}])}{dt} = F_{in} c_v (T_{in} - T_{ref}) - F_{out} c_v (T - T_{ref}) + Q_{steam} - Q \]  

(9)

\[ \rho c_p A_v \frac{d(h[T - T_{ref}])}{dt} = F_{in} c_v (T_{in} - T_{ref}) - F_{out} c_v (T - T_{ref}) + Q_{steam} - Q \]  

(10)

Since \( T_{ref} \) is constant, Equation (10) reduces to

\[ \rho c_p A_v \frac{d(T)}{dt} = F_{in} c_p (T_{in} - T_{ref}) - F_{out} c_p (T - T_{ref}) + Q_{steam} - Q \]  

(11)

\[ \rho c_p A_v \frac{dT}{dt} + \rho c_p A_v \frac{dh}{dt} = F_{in} c_p (T_{in} - T_{ref}) - F_{out} c_p (T - T_{ref}) + Q_{steam} - Q \]  

(12)

\[ \rho c_p A_v \frac{dT}{dt} = F_{in} c_p (T_{in} - T_{ref}) - F_{out} c_p (T - T_{ref}) + Q_{steam} - Q - c_p T \left( \rho A \frac{dh}{dt} \right) \]  

(13)

From Equation (6),

\[ \rho A \frac{dh}{dt} = F_{in} - c_v \sqrt{\rho gh} \]  

(6)

Therefore, Equation (13) becomes,

\[ \rho c_p A_v \frac{dT}{dt} = F_{in} c_p (T_{in} - T_{ref}) - F_{out} c_p (T - T_{ref}) + Q_{steam} - Q - c_p T \left( F_{in} - c_v \sqrt{\rho gh} \right) \]  

(14)

\[ \rho c_p A_v \frac{dT}{dt} = F_{in} c_p (T_{in} - T_{ref}) - F_{out} c_p (T - T_{ref}) + Q_{steam} - Q - c_p T F_{in} + c_p T c_v \sqrt{\rho gh} \]  

(15)

\[ \rho c_p A_v \frac{dT}{dt} = F_{in} c_p (T_{in} - T_{ref}) - F_{out} c_p (T - T_{ref}) + Q_{steam} - Q + c_p c_v T \sqrt{\rho gh} \]  

(16)

\[ \rho c_p A_v \frac{dT}{dt} = F_{in} c_p (T_{in} - T_{ref} - T) - (c_v \sqrt{\rho gh}) c_p (T - T_{ref}) + Q_{steam} - Q + c_p c_v T \sqrt{\rho gh} \]  

(17)
\[
\rho c_p A_h \frac{dT}{dt} = F_w c_p (T_{in} - T_{ref} - T) - c_p c_v T \sqrt{\rho g h} + c_p c_v T_{ref} \sqrt{\rho g h} + Q_{steam} - Q + c_p c_v T \sqrt{\rho g h} \tag{18}
\]

\[
\rho c_p A_h \frac{dT}{dt} = F_w c_p (T_{in} - T_{ref} - T) + c_p c_v T_{ref} \sqrt{\rho g h} + Q_{steam} - Q \tag{19}
\]

\(Q_{steam}\) is the heat transferred from steam to liquid, and \(Q\) is the heat lost as a result of the vaporization of the liquid, and they are given as:

\[
Q_{steam} = (UA)_{col} \frac{h}{h_{max}} (T_{steam} - T) \tag{20}
\]

and

\[Q = F_{in} \Delta H_{vap} \tag{21}\]

Substituting the expressions for \(Q_{steam}\) and \(Q\) into Equation (19),

\[
\rho c_p A_h \frac{dT}{dt} = F_w c_p (T_{in} - T_{ref} - T) + c_p c_v T_{ref} \sqrt{\rho g h} + (UA)_{col} \frac{h}{h_{max}} (T_{steam} - T) - F_{in} \Delta H_{vap} \tag{22}
\]

\[
\frac{dT}{dt} = \frac{F_w c_p (T_{in} - T_{ref} - T) + c_p c_v T_{ref} \sqrt{\rho g h} + (UA)_{col} \frac{h}{h_{max}} (T_{steam} - T) - F_{in} \Delta H_{vap}}{\rho c_p A_h} \tag{23}
\]

Equations (7) and (23) are the two (nonlinear) theoretical models of the evaporator. The models can, actually, be used to study the dynamics of the system. However, in order make it easier for the dynamic simulation of the system to be carried out, and for the purpose of deriving a system of models that can be used, conveniently, to carry out the control of the system, its transfer function models were developed via its state space form.

The state space form of the model of the system is given as:

\[
\dot{x} = Ax + Bu
\]

\[
y = Cx + Du \tag{24}
\]

where \(x\) is the state, \(u\) is the input and \(y\) is the output variables of the system. Also, coefficients \(A, B, C\) and \(D\) are the state matrix, the input matrix, the output matrix and the feedthrough matrix, respectively.

The values of \(A, B, C,\) and \(D\) matrices of the state-space model of the system were estimated with the aid of MATLAB, and, thereafter, it was converted into state-space form recognized by MATLAB using the ss command of MATLAB. Then, the transfer function models of the system were obtained using the ss2tf command of the same MATLAB, and their values were entered into the transfer function blocks of the Simulink model of the system shown in Figure 2, for simulation. Simulink was used because it has been discovered to be effective and convenient in simulating Chemical Engineering processes. For instance, it has been used successfully for Chemical Engineering process simulation in the work of Giwa and Karacan (2012a, b).

As can be seen from the developed Simulink model of the system, the transfer function model developed is a multi-input multi-output (MIMO) type that has two inputs (feed flow rate and steam temperature) and two outputs (evaporator liquid height and exit temperature).

The data that were used to carry out the simulations involved in this work are as given in Table 1 below.
The simulation of the Simulink model of the system, shown in Figure 2, was carried out using the \textit{sim} command of MATLAB, and the results obtained from the simulations, together with the results of the other simulations carried out are as given and discussed in the next section.

![Simulink model](image)

**Figure 2. The Simulink representation of the transfer function model of the system**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial feed flow rate, $F_{in}$</td>
<td>0.5 kg/min</td>
</tr>
<tr>
<td>Valve constant, $c_v$</td>
<td>1.1 kg/(min bar$^{0.5}$)</td>
</tr>
<tr>
<td>Liquid density, $\rho$</td>
<td>800 kg/m$^3$</td>
</tr>
<tr>
<td>Acceleration due to gravity</td>
<td>9.81 m/s$^2$</td>
</tr>
<tr>
<td>Evaporator cross-sectional area, $A_c$</td>
<td>0.3 m$^2$</td>
</tr>
<tr>
<td>Inlet temperature, $T_{in}$</td>
<td>300 K</td>
</tr>
<tr>
<td>Product of the heat transfer coefficient and heat transfer area, $(UA)_{coil}$</td>
<td>12.6 kJ/(min K)</td>
</tr>
<tr>
<td>Specific heat, $c_p$</td>
<td>6 kJ/(kg K)</td>
</tr>
<tr>
<td>Maximum liquid level, $h_{max}$</td>
<td>1 m</td>
</tr>
<tr>
<td>Steam temperature, $T_{steam}$</td>
<td>600 K</td>
</tr>
<tr>
<td>Heat of vaporization, $\Delta H_{vap}$</td>
<td>463 kJ/kg</td>
</tr>
<tr>
<td>Reference temperature, $T_{ref}$</td>
<td>298 K</td>
</tr>
</tbody>
</table>

**Table 1. Evaporator system simulation data**

**Result and Discussion**

The results obtained from the dynamic simulation of the evaporator when the values of the input variables, liquid feed flow rate and steam temperature were fixed at 0.5 kg/min and 600 K respectively are as shown in Figures 3 – 4.
below. Figure 3 shows the dynamic response of the liquid height while Figure 4 contains the dynamic response of the evaporator system temperature.

![Graph showing dynamic response of evaporator liquid height](image)

**Figure 3.** Dynamic response of evaporator liquid height (for $F_{in} = 0.5$ kg/min and $T_{steam} = 600$ K)

As can be seen from the figures, in responding to the input values used for the simulation, the liquid height decreased with time until it got to its steady state value of approximately 0 m (2.6327e-05 m) while the system temperature first increased, but later decreased to attain its steady-state value of approximately 224.02 K.

The steady-state value obtained for the liquid height implied that at the end of the evaporation operation, no liquid was left in the evaporator. Well, this might be so because it was possible for all the liquid present in the evaporator to turn to vapor. However, the value obtained for the steady-state of the system temperature was found not be realistic because that value was discovered to be below the freezing point of water. Even though, the liquid being evaporated was not mentioned, at least, it was mentioned that steam (with freezing point = 273 K) was used to carry out the evaporation. As such, if the 224.02 K obtained as the steady-state value of the system temperature were to be realistic, it implied that ice would have been formed in the steam line being used for the evaporation operation at that time. As such, the obtained steady-state values were found not be reliable, and this necessitated using another method to obtain the steady-state values of the output variables (liquid height and system temperature).
In view of the unrealistic nature of the steady-state values obtained from the dynamic simulation, the dynamic model of the system was solved for their steady-state values with the aid of \texttt{fsolve} command of MATLAB using function evaluation limit of 200, and the values that were obtained were found to be 0.51547 m and 1261.3 K respectively for the liquid height and the system temperature. According to these steady-state values obtained for the system, it has been discovered that, at the steady state of the evaporation operation, approximately 0.52 m of liquid was remaining in the evaporator at the temperature of 1261.3 K. These values were, actually, found to be more reasonable and justifiable because it implied that the initial liquid height of the evaporator had been reduced by approximately 0.48 m, and that the accumulation of heat in the evaporator, which was the concept used to develop the dynamic model of the system, had resulted in increase in the temperature of the system from the initial value of 298 K to 1261.3 K.

Having obtained realistic steady-state values of the system, they (the steady-state values) were used to obtain the state-space form of the model of the system. The state-space model of the system obtained, with the variables in their deviation variable forms, is given as shown in Equation (25).

\[
\begin{bmatrix}
\dot{h}(s) \\
\dot{T}(s)
\end{bmatrix} =
\begin{bmatrix}
-0.2828 & 0 \\
-153 & -0.2506
\end{bmatrix}
\begin{bmatrix}
h(s) \\
T(s)
\end{bmatrix}
+ 
\begin{bmatrix}
-0.004167 & 0 \\
-10.8 & 0.2462
\end{bmatrix}
\begin{bmatrix}
F_{in}(s) \\
T_{steam}(s)
\end{bmatrix}
\]

(25)

Using the state-space form of the system model, the transfer function model of the system was thus, also, estimated to be as given in Equation (26).

\[
h(s) = \frac{0.004167s + 0.001044}{s^2 + 0.5333s + 0.07085} F_{in}(s) + 0 \cdot T_{steam}
\]

(26)

\[
T(s) = \frac{-10.803s - 3.692}{s^2 + 0.5333s + 0.07085} F_{in}(s) + \frac{0.2465s + 0.06971}{s^2 + 0.5333s + 0.07085} T_{steam}
\]
It was very clear from the transfer functions obtained that the model of the system was a second order type because the denominator of each of the terms present in the transfer function model was quadratic.

Now, simulating the transfer function models obtained by applying step changes to the input variables (liquid feed flow rate and steam temperature) to obtain the response of the output variables (liquid height and system temperature), the results obtained are as given in Figures 5 – 10.

In Figure 5, the dynamic response of the liquid height when a +0.2 kg/min step change (from steady state) was applied to the liquid feed flow rate at time $t = 1\ min$ is shown. As can be seen on the graph, a positive step change in the liquid feed flow rate of the system has resulted in a positive step move of the liquid height to another steady state.

Figure 5. Dynamic response of liquid height to a +0.2 kg/min step change in feed flow rate

Figure 6 shows the dynamic response of the system temperature upon applying a +0.2 kg/min step change to feed flow rate at time $t = 1\ min$. In this case, a negative step change was noticed in the system temperature as a result of the positive change in the feed flow rate.
From Figures 5 and 6, it has been discovered that an increase in the feed flow rate was able to result in an increase in the level of the liquid in the evaporator, but this caused a decrease in the temperature of the system because more liquid was passed into the system to compete in getting heated and evaporated with the steam being passed but the flow rate and the temperature of which were not increased to cater for the increase in the amount of the liquid present in the system at that particular time.

Figure 7. Dynamic response of liquid height to a +10 K step change in steam temperature

Given in Figure 7 is the dynamic response of liquid height to a +10 K step change in steam temperature. As can be seen from the graph, the step change in the steam temperature did not bring about any change in the liquid height of...
the evaporator. However, in Figure 8 that is showing the dynamic response of the system temperature to a +10 K step change in steam temperature, a positive change was noticed to occur. That is, an increase in the temperature of the steam being passed caused the temperature of the system to also increase.

![Figure 8. Dynamic response of system temperature to a +10 K step change in steam temperature](image)

Considering the observations made from the results obtained when a step change was applied to the liquid feed flow rate (Figures 5 and 6) and those obtained when a step change was applied to the steam temperature (shown in Figure 9).

![Figure 9. Dynamic response of liquid height to +0.2 kg/min and +10 K step changes in feed flow rate and steam temperature, respectively](image)
Figures 7 and 8), it has been discovered that feed flow rate was alone able to have effects on the two output variables (liquid height and system temperature) of the system while the steam temperature was alone able to produce a change in the system temperature only.

In order to further investigate how both the liquid feed flow rate and the steam temperature (the input variables of the system) affect the operation of the evaporator, that is, to know their combined effects of the liquid height and the system temperature (output variables of the system), steps were applied to both of them simultaneously, and the system was simulated in Simulink environment. The results obtained from this simulation of the Simulink model of the system, upon the application of steps to the two input variables, are as given in Figures 9 and 10.

![Figure 10. Dynamic response of system temperature to +0.2 kg/min and +10 K step changes in feed flow rate and steam temperature, respectively](image)

Based on the results given in Figures 9 and 10, it was discovered that the application of step changes to the two input variables simultaneously was able to show effects on the output variables of the evaporator because a step change of +0.2 kg/min in the liquid feed flow rate and a step change of +10 K in the steam temperature was able to produce changes in the responses of both the liquid height and the temperature of the system.

**Conclusions**

The results obtained from the development of the transfer function model of the evaporator system studied in this work have revealed that the steam temperature had not any effect on the liquid level of the system because the coefficient of the transfer function relating the liquid level of the evaporator to the steam temperature was zero. This was later observed when the Simulink model of the system was simulated as the application of a step to steam temperature resulted in no change in the steady-state value of the liquid level. It was also discovered that a step change in the liquid feed flow rate of the system affected both the liquid level and the temperature of the evaporator. As such, even though a step change in the steam temperature was able to affect only the temperature of the evaporator, the system has been discovered to be a multi-input multi-output (MIMO) type because, according to the dynamic responses obtained from the system, one of the input variables (liquid feed flow rate) affected the two output variables (liquid height and system temperature) of the system.
Nomenclature

Notations

\( A_c \) & Evaporator cross-sectional area, \( m^2 \) \\
\( c_p \) & Specific heat capacity, \( KJ/(kg \ K) \) \\
\( c_v \) & Valve constant, \( kg/(min \ bar^{0.5}) \) \\
\( F_{in} \) & Feed flow rate, \( kg/min \) \\
\( F_{out} \) & Exit flow rate, \( kg/min \) \\
\( g \) & Acceleration due to gravity, \( m^2/s \) \\
\( h \) & Liquid level, \( m \) \\
\( P \) & Vapor pressure, \( bar \) \\
\( P_{out} \) & Outlet pressure, \( bar \) \\
\( Q \) & Heat transfer due to vaporization, \( kJ/min \) \\
\( Q_{steam} \) & Heat transfer from steam to liquid, \( kJ/min \) \\
\( t \) & Time, \( s \) \\
\( T \) & System temperature, \( K \) \\
\( T_{steam} \) & Steam temperature, \( K \) \\
\( T_{ref} \) & Reference Temperature, \( K \)

Greek letter

\( \rho \) & Liquid density, \( kg/m^3 \)

References


